Currency Invoicing and Risk

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Abstract

Firms take on risk when they trade internationally. Payments and shipments for goods are often lagged up about 90 days, and between firms there is always the chance that one firm breaks contract. We developed a payment choice model within a two-country dynamic general equilibrium framework to draw out the consequences of these two frictions. Our model shows that, when home interest rates are high, exports will be invoiced in the low interest rate foreign currency. This will lead to greater real volatility in consumption and, therefore, higher precautionary savings by home consumers. Our preliminary findings are broadly consistent with the data.

1 Introduction

It can be risky to trade goods across borders. Firms can experience long delays in shipping times and currency fluctuations can degrade the value of the product before it arrives, if it ever does. As the volume of global trade is large and increasing, countries are depending evermore on trade flows for income and consumption. In this paper we developed a currency and payment choice model within a standard two-country dynamic general equilibrium framework. We focus on the way that importers and exporters pay for goods across borders and show that it is directly linked to the currency of invoicing and the hoarding of precautionary savings.

Across time, most international payments and shipments are lagged about 90 days, and sometimes longer when the partners are far from each other. The impact of this delay is significant. Amiti and Weinstein (2011) discuss how firms that trade internationally and are subject to delays that often extend beyond 90 days, are also the firms that suffer the greatest during a liquidity crises. Leibovici and Waugh (2019) show that, by including such a delay, they can rectify two nagging problems with DSGE models: over-predictions of the price elasticity of trade and under-predictions for income elasticity of imports. In this model, international shipments will be delayed one period. Risk sharing is assumed to be incomplete, so the result is that net exports play a larger role in risk sharing, usually being a source of volatility.
In light of shipping times, importers and exporters can agree on one of several ways for the importer to pay for the good. Most commonly used are Open Account (OA) transactions, whereby the importer pays the exporter after the good arrives. Also, there are Letters of Credit (LOC) transactions, where the importer’s bank pays the exporters bank upon arrival of the good. Finally, firms may require a Cash-in-Advance transaction when the importer pays for the product before it arrives.

Between firms there is always the chance that one firm fails to deliver, i.e. breaks contract. We show that the cost of breaking contract varies dramatically across countries, and that firms probably consider contract enforcements costs when they choose a settlement method. Both Niepmann and Schmidt-Eisenlohr (2013) and Schmidt-Eisenlohr (2013), show this to be the case, however, our paper extends frictions to a dynamic general equilibrium framework to derive conclusions about net export and savings behavior.

Specifically, we put two frictions, lagged shipments across time and contract enforceability between firms, into a standard DSGE framework. Our model shows that, when home interest rates are high, exports will be invoiced in the low interest rate foreign currency. This will lead to greater real volatility in consumption and, therefore, higher precautionary savings by home consumers. These predictions are broadly consistent with the data and other research. Gopinath (2016) calls attention to what she coins the ‘International Pricing System’: firms from all countries overwhelmingly invoice in either the U.S. dollar or the Euro, even those firms that reside in countries that do not use the Euro or Dollar as a domestic currency. This means that payments for traded goods have significant exchange rate risk. As more trade is invoiced in foreign currency, aggregate price volatility increases as does the correlation of prices with exchange rate movements. For some countries, as much as 50% of GDP is trade denominated in a foreign currency.

Fogli and Perri (2015) establish a link between real volatility and savings by showing that countries with increasing volatility in output and wages also have increasing foreign assets. One contribution of this chapter is to link these two mechanisms: currency invoicing and savings.

This paper can offer a partial explanation for the buildup of foreign exchange reserves in developing countries, but we are modest about the model’s performance against the data. A common observation is that most of the Net Foreign Asset (NFA) surpluses in developing countries are driven by public savings, or more specifically, by a buildup in foreign exchange reserves. Alfaro, Kalemli-Ozcan, and Volosovych (2008) establishes that on average fast growing countries run current account surpluses, but this trend is purely driven by public flows. Private flows conform with

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1Japan and Turkey, for example, have about 10% of all imports invoiced and priced in their home currency, leaving about 80% priced in U.S. dollars. In terms of pass-through, a 10% change in the U.S. Dollar will translate into a 2% change in the aggregate price levels in Turkey and similarly for Japan. For the U.S. however, which denominates 90% of all trade in its home currency, the same 10% change will only move the U.S. aggregate price level by 0.3%. Since nominal exchange rates are notoriously volatile, aggregate price levels and firms profits in Japan and Turkey will be more volatile relative to the U.S.
Figure 1: Trade invoiced in a Foreign Currency as a Share of GDP

(a) This graph shows the volume of trade invoiced in a foreign currency as a share of GDP. This data is compiled from a variety of sources, please see appendix for details.

This paper is most related to the literature that links financial frictions and trade flows. Most of this research focuses on how financial frictions affect trade. Since trade takes time and is risky, firms depend on working capital loans to provide them liquidity. During tight credit conditions, there is an ‘acceleration’ mechanism that constrains firm output. Amiti and Weinstein (2011) establish that bank health and trade finance is an important determinant for firm-level exports during crisis. Since (1) 90% of all firms engage in some sort of trade credit, and (2) a firm’s payables/receivables may account for as much as 30% of a firm’s total revenue, then firms depend on working capital loans and any hit to bank lending will also hit international trade. Ahn, Amiti, and Weinstein

2This is a bold assumption, but it is not a ridiculous one. There are many examples where sovereign reserves are lent out during bad times to domestic investors and consumers, just as a representative consumer uses their savings in bad times to smooth consumption. The governments of Korea and Indonesia during the 1998 Financial Crisis, or Brazil between 2002-2003 are two examples discussed in Wang and Ronci (2006), and Chauffour and Farole (2009) survey several more recent incidences from the 2008 Financial Crisis.
This paper is related to the vast literature on ‘global imbalances’, in which most papers focus on current account imbalances and NFA positions with some focus on portfolio structures. Mendoza, Quadrini, and RíosRull (2009) argue that persistent imbalances exist because financial markets are heterogeneous across countries, and this may cause some countries to act more risk-averse than others. Caballero, Farhi, and Gourinchas (2006) models a world in which a country with lower financial risks are better able to supply financial assets, hence generating the current imbalances in the data. In a similar paper that instead focuses on sudden stops, Korinek and Mendoza (2014) use a Fisherian debt-deflation mechanism. Finally, in two papers that focus on exchange rate risk, Maggiori et al. (2011) and Gabaix and Maggiori (2014) present similar models where financial flows
are intermediated by a global financier who must be compensated for holding the currency risk in the form of an expected currency appreciation. The risk bearing capacity (or degree of financial integration) determines expected returns and distorts exchange rates.

Several papers test the determinants of a currency invoicing, but do not consider the method of pricing (i.e. OA, CIA, LOC) or the impact on savings behavior. Goldberg and Tille (2009) apply a game theoretic model to a firm level dataset of Canadian importers and examine the determinants of invoicing choices. Ito and Kawai (2016) develop upon a dataset that focuses on the experiences of the U.S. dollar, the Japanese Yen and the Deutsche Mark in the 1970’s through to the 1990’s, and develop an empirical model and project the success of the Chinese renminbi. Hiroyuki, Masahiro, et al. (2016) build further on that dataset to project the future success of the renminbi as an invoicing currency.

The next section of this paper will discuss the model. Section 3 explains briefly the numerical solution method, Section 4 summarizes the results and Section 5 concludes.

2 A Model of Payment & Currency Choice

The model starts from a standard two-country dynamic general equilibrium setup, but in solving the model, we set the foreign countries consumption and inflation as exogenous. There are two productive industries within each country: one produces strictly non-traded goods, while the other produces traded goods that are only consumed abroad. The non-traded goods sectors is perfectly competitive with perfectly flexible prices. Our focus will be the traded goods sector that will consist of three agents, each representing a vital step in the changing-of-hands for international shipments. The first agent is the Exporter. They produce the homogeneous product using local labor and ship the product abroad to the receiving Importer. Importers bear the brunt of risk from currency fluctuations and contract disputes. In turn, the Importers deliver the product to Retailers that sell at market clearing prices to consumers. This model will focus on the interactions of those three agents: Export, Importer and Retailer.

There are three ways that the Exporter can sell to the Importer. The most common way, at least among developed countries, is to use Open Account payments whereby the importer pays for the product after its delivery. They may do this with varying degrees of lateness, not often exceeding 90 days, but usually surpassing 45. Arguably, the second most common way that firms pay for goods is by using Cash-In-Advance payments. Here, the importer pays before the good before it is shipped (or produced at all). Finally, financial instruments provided by banks can be used to settle payments. Most commonly used instruments are the Letters-of-Credit (LOC) and Documentary Collections (DC) whereby the importer’s bank pays the exporter upon receipt of the good. Payment for traded goods can then be settled before, during, or after receipt. Also, importantly, payments
for traded goods can be settled in either the home or foreign currency. In this model, firms choose
the currency of settlement contemporaneously with the delay of settlement to maximize profits.

As mentioned, there are two key frictions in this model. First, there is a one-period delay in the
arrival of traded goods. This is convenient from a modeling standpoint and it helps to reconcile the
income elasticity of imports with the data. Second, exporting and importing firms are heterogeneous
in the degree that shipping contracts can be enforced. We assume that each firm $i$ starts with a $\delta_i$
that represents the cost of enforcing a contract as a percentage of the shipment.

### 2.1 Preferences

The Home (H) and Foreign (F) countries produce and trade one manufactured good, and bonds
denominated in the foreign currency. Consumers in the home country consume traded ($C_{T,t}$) and
non-traded ($C_{N,t}$) goods. The world economy is populated with a continuum of agents where the
population indexed in $[0,n]$ live in $H$ and everyone else lives in $F$. Consumers aggregate over the
two goods using a Dixit-Stiglitz-type aggregator.

$$C_t = \left( \nu^{1/\sigma} (C_{T,t})^{(\sigma-1)/\sigma} + (1 - \nu)^{1/\sigma} (C_{N,t})^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$$

where $\sigma$ is the elasticity of substitution between home and foreign goods, and $\nu$ is the share of
foreign goods in consumption, but can be interpreted as a function of the relative size of $H$ to $F$.
We define $\nu = n \xi$ where $\xi$ is the degree of openness. As we take $n \to 1$, the model approaches a
small-open economy where $\lambda \to \nu$. Foreign consumers have a similar expression for consumption,
except now $\nu^* = (1 - n) \xi$ so the role of home goods in $F$ becomes negligible.

Price indices across traded and non-traded goods are standard. We normalize all prices
by the national price index, so that all prices are in terms of the composite good at home
and abroad, $C_t$, and $C^*_t$, respectively. Prices for traded and non-traded goods must satisfy,
$$1 = \left( \nu P_{T,t}^{1-\sigma} + (1 - \nu) P_{N,t}^{1-\sigma} \right)^{1/(1-\sigma)},$$
where the aggregate price index $P_t$ has been divided into the index, so that all prices are in terms of the final good. Foreign prices must satisfy a similar condition.
Consumers have standard CRRA preferences over the composite good, where $\gamma$ is the coefficient
of relative risk aversion. Providing a unit of labor $L_t$ yields dis-utility according to the parameter
$\eta$. Finally, consumers enjoy holding real cash balances for its liquidity properties, and they choose
to hold $M_t/P_t = \tilde{M}_t$, where $P_t$ is the aggregate price index that has been divided into all nominal
variables, and it will therefore not appear in our equilibrium equations. The parameter $\epsilon$ is the
elasticity of real money balances demand and $\varrho$ is a parameter (set close to zero) that converts real
balances to utility units.

\[ U \left( C_t, \bar{M}_t, L_t \right) = \frac{C_t^{1-\gamma}}{1-\gamma} - \eta L_t + \frac{\varrho}{1-\epsilon} \bar{M}_t^{1-\epsilon} \] (2)

Money is supplied by a simple government that makes lump sum transfers (or withdraws) of real balances, \( \tau_g^t \), according to

\[ \tau_g^t = \bar{M}_{t+1} - \bar{M}_t \] (3)

Here we define the inflation rate, \( \Pi_{t+1} = P_{t+1}/P_t \). The government can then control money growth through \( \tau_g^t \) to change the inflation rate. Although prices are perfectly flexible, there are delays in shipments. The relative price changes of traded vs. non-traded goods will leave room for the inflation rate to affect real quantities and welfare.

Consumers at home can buy non-contingent nominal bonds in either currency (\( H \) or \( F \)), however, only bonds in the foreign currency are traded internationally. Home bonds are sold at \( Z_{h,t} \) and foreign ones are sold at price \( Z_{f,t} \). A bond price is just the inverse of the interest rate minus 1, or \( Z_{j,t} = 1/(1+r_{t,j}) \). We will use the two forms of bond valuation interchangeably throughout this paper.

The consumer supplies labor, \( L_{i,t} \), at real wage rate \( W_{i,t} \). Profits will be discussed later, but for now \( \Pi_{T,t} \) will be the sum of profits from the Retailers, Importers and Exporters in the home country. Combining the government budget constraint in Equation 3, with all other income and expenses of the consumer, we arrive at the home consumer’s budget constraint.

\[ C_{h,t} + Z_{h,t}B_{h,t+1} + Z_{f,t}Q_tB_{f,t+1} + \bar{M}_{t+1} \leq W_{h,t}L_{h,t} + \frac{B_{h,t}}{\Pi_t} + \frac{Q_t}{\Pi_t}B_{f,t} + \Pi_{T,t} + \frac{\bar{M}_t}{\Pi_t} + \tau_t \] (4)

All variables here are expressed in real terms, and will be for the remainder of the paper. \( Q_t \) is the real exchange rate defined as \( S_t \) is the nominal exchange rate defined as \( Q_t = S_tP_{t}^{*}/P_t \), where \( S_t \) is the nominal exchange rate. In this set up, an increase in \( S_t \) is a depreciation in the nominal currency. The consumer is confronted with maximizing their lifetime utility (2), subject to their budget constraint (4), and their preferences across goods (1). With discount factor \( \beta \), consumers solve,

\[
\max_{C_t, B_{t+1}, B_{f,t+1}, L_{h,t}, \bar{M}_t} \left\{ E_t \sum_{t=s}^{\infty} \beta^{t-s} U \left( C_t, \bar{M}_t, L_t \right) \right\} \\
\text{s.t. } (1), (2), (4)
\]
2.2 Consumer Choices

The first order conditions for the consumer problem yield familiar expressions for the demand of traded and non-traded goods: respectively 

\[ C_{T,t} = \nu (P_{T,t})^{-\sigma} C_t, \]  

and 

\[ C_{N,t} = (1 - \nu) (P_{N,t})^{-\sigma} C_t. \]

The foreign consumer will have similar expressions. To make the exposition easier, we define the stochastic discount factor as \( \Delta_{t+1} \equiv (C_{t+1}/C_t)^{-\gamma} \). Prices for home and foreign bonds satisfy

\[
Z_{h,t} = \beta E_t \left[ \frac{\Lambda_{t+1}}{\Pi_{t+1}} \right] \\
Z_{f,t} = \beta E_t \left[ \frac{\Lambda_{t+1} Q_{t+1}}{\Pi_{t+1} Q_t} \right] \\
\hat{Z}_{h,t} = \beta E_t \left[ \frac{\Lambda^*_t Q_t}{\Pi_{t+1} Q_{t+1}} \right]
\]

(5a)

(5b)

Notice that Equation 5b for \( \hat{Z}_{h,t} \) is slightly different from the others. This is the the price of a bond that is never traded, i.e. it is the price a consumer in F would pay for \( B_{h,t+1} \). We define the hypothetical price because it helps later when discussing producer decisions. This bond is not traded because we want to limit the scope of the model: having only one traded bond (\( B_{f,t+1} \)) aids greatly in tractability as it avoids solving a portfolio choice problem. Qualitatively, the results are likely to be similar when consumer’s trade bonds of all currencies. Also, the results show that \( Z_{h,t} \) tracks \( \hat{Z}_{h,t} \) quite closely for most states of our model world. Demand for real balances at home yields an additional equation for interest rates.

\[ Z_{h,t} = 1 - \varrho \left( \tilde{M}_t \right)^{-\epsilon} C_t^{\gamma} \]

Consumers choose labor \( L_t \) to satisfy the first order conditions from the consumer problem: \( W_t = \eta C_t^{\gamma} \). The next section describes the behavior of the Retailer.

2.3 Retailers

Consumers buy traded goods from one of a continuum of perfectly competitive retailers. Each retailer is owned by the consumer, so any returns on their operations are discounted stochastically by \( \Delta_{t+1} \), and paid back the consumer as a lump sum (as discussed below, Retailer profits are included in \( \Pi_{T,t} \)). At the start of each period, after uncertainty in the exogenous shocks have been resolved but before consumers choose \( L_t \), Retailers are tasked with finding an importer and purchasing (a share of) \( C_{T,t+1} \) to be delivered next period. We assume all goods are perishable and so abstract away from a discussion about inventories.

Each period, the retailer demands a quantity \( C_{t+1} \) for sale next period. Importers and Retailers decide on the price to be paid next period, \( P_{t,t} \). The firm maximizes the expected value of profits
that they will earn next period from the goods they purchase this period.

$$\max_{C_t} E_t \left\{ \Delta_{t+1} \left( CT_{t+1} PT_{t+1} - \frac{PT_{t+1} CT_{t+1}}{\Pi_{t+1}} \right) \right\}$$

The price $P_{T,t}$ must clear the markets, so we know that

$$P^*_t = \left( \frac{C^*_t}{C^*_{t+1}} \right)^{-1/\sigma}$$

Updating this expression and plugging this back into the first order condition for the retailer, yields the final demand for traded goods arriving next period

$$C^*_{T,t+1} = \nu C^*_t \gamma \sigma \left( \frac{P^*_t Z_{t,t}}{E_t \left[ \frac{C^*_t}{\Pi_{t+1}} \right]} \right)^{-\sigma} \tag{6}$$

It is important to note that $CT_{t+1}$ and $PI_{t,t}$ are known with certainty at time $t$. The price of traded goods, $PT_{t+1}$ is unknown at $t$ because these prices depend on the relative valuation of traded and non-traded goods (embedded in $C_{t+1}$). Demand for traded goods in Equation 6 makes sense. Future traded goods consumption are increasing with past consumption and increasing in the domestic interest rate. Also, as long as the agent is sufficiently risk averse, demand for traded goods this period increases when expected consumption next period is low. Traded goods can behave like a savings instrument, helping to deliver consumption next period when it is more highly valued.

### 2.4 Non-Traded Goods

In the non-traded goods industry, producers solve a simple production problem.

$$\max_{C_{N,t}} \{ C_{N,t} (P_{N,t} - W_t) \}$$

There are no barriers to entry and the product is uniform, so firm profits are zero and prices become $P_{N,t} = W_t$. 
2.5 Shocks

Shocks to the system come from the foreign country. We assume that the foreign country’s consumption and inflation each follow an exogenous forcing process,

\[ C_t^\star = \rho_c^\star C_{t-1}^\star + (1 - \rho_c^\star) \bar{C}^\star + \epsilon_c^t \]
\[ \Pi_t^\star = \rho_{\pi}^\star \Pi_{t-1}^\star + (1 - \rho_{\pi}^\star) \bar{\Pi}^\star + \epsilon_{\pi}^t \]

where \( \rho_c^\star < 1 \) and \( \rho_{\pi}^\star < 1 \) are the persistence of shocks in foreign consumption and inflation, respectively. The shocks to each autocorrelated series, \( \epsilon_c^t, \epsilon_{\pi}^t \), are distributed \( N(0, \sigma_c) \) and \( N(0, \sigma_{\pi}) \), respectively. The next section summarizes the optimal behavior of the consumer.

2.6 Exporters and Importers

Similar to the non-traded goods sector, producers of traded goods will produce using constant returns to scale. There are, however, three significant differences between these sectors. **First**, each unit of output must be accompanied by some cost if the contract is broken. These costs, summarized in what we call a reneging factor \( \delta_i \), follows a distribution unique to each country. If a firm reneges, portion \( \delta_i dC_{T,t+1}^i \) is lost in court costs. The random variable will be distributed in the range \( [0, 1/u] \), where \( u \) is the average number of times per period a firm reneges on a contract. Foreign importers/exporters likewise have a \( \delta_i^\star \) that follows the same distribution. **Second**, as we mentioned, international shipments are delayed in arrival by one period after ordering the product so that aggregate demand follows Equation 6. **Finally**, to mitigate the risk from the delay in shipments, Importers and Exporters have three methods to transmit payments for international goods: Cash-In-Advance (CIA), Letters of Credit (LOC), and Open Account (OA). Firms can also choose the currency, Home (H) or Foreign (F) in which to settle the payment, so that in total there are six discrete methods of payment (3 methods x 2 currencies): \( CIA_h, CIA_f, OA_h, OA_f, LOC_h, \) and \( LOC_f \).

At the start each period, agents observe exogenous shocks to foreign consumption and inflation, \( \epsilon_c, \epsilon_{\pi} \). Exporters pull from the continuous distribution of \( \delta_i \)’s, and importers do the same from their own distribution of \( \delta_i^\star \). Exporters and importers are then randomly matched until there are none remaining, so that there is a joint probability distribution of matched firms, \( f_{\delta\delta^\star}(\delta_i^\star, \delta_i^\star) = f_\delta(\delta_i) f_{\delta^\star}(\delta_i^\star) \) with the CDF of the joint distribution given by \( F_{\delta\delta^\star}(\delta_i^\star, \delta_i^\star) \). Both importers and exporters have full information about each others’ reneging factor, and they can either agree on one of the six payment type mentioned, or they can decide not to produce that period.
2.7 Distribution of $\lambda$

In this section, we motivate the random variable $\lambda_{i,t}$ as the percentage left over after reneging on a contract, and justify our choice of PDF for $\lambda_{i,t}$ using data on contract enforcement. We define,

$$\lambda_{i,t} = 1 - \delta_i \mu_t$$  \hspace{1cm} (7)

The arrival of some default shock, $\mu_t$, is distributed $Bernoulli(u)$ with $f_u(\mu_t)$ as the PDF,

$$f_u(\mu_t) = \begin{cases} 
    u & \mu_t = 1 \\
    1 - u & \mu_t = 0 
\end{cases}$$

In the event of a default ($\mu_t = 1$), share $\delta_i$ of the shipping contract is lost. The default shock ($\mu_t$) is idiosyncratic to the firm, but identically distributed across all firms. The cost of default ($\delta_i$) follows a time-invariant distribution indexed by each shipment $i$. We assume that $\mu_t$ is uncorrelated with any other variables in the model. Of course, the model becomes very interesting but quickly loses tractability when we allow for covariances between $\mu_t$ and endogenous variables.

The distribution of $\delta_i$ can be motivated by Figure 3, showing data from the World Bank’s *Doing Business Survey*. The data show the cost of enforcing contracts as a percent of the claim. These data are then weighted by their share of trade over total average world trade and recast as a density in Figure 3. For robustness we include two measures to capture the costs of contract enforcement, the first being the ‘cost of contract enforcement’ as a percent of the claim. Second is the percent of debt recovered when the firm becomes insolvent$^3$. One caveat here is that neither series is exclusively for firms that trade internationally, which may be problematic as trading firms often differ from those that just operate domestically.

To approximate the density in Figure 3, a tractable function that is commonly used in models with heterogeneous firms$^4$ is the Pareto distribution. If $u$ is the expected value of a crisis, and $z_i$ is distributed $Pareto(1, \kappa)$, then we define,

$$\delta_i = \frac{1}{u} \left( 1 - \frac{1}{z_i} \right)$$  \hspace{1cm} (8)

A variable $z \sim Pareto(\hat{z}, \kappa)$ has PDF $f(z) = \kappa \hat{z}^{\kappa} z^{-\kappa - 1}$, where $\hat{z}$ is the minimum value in the distribution. Re-arranging Equation 8 and using this definition, the PDF for $\delta_i$ will be

$$f(\delta_i) = k \left( 1 - u\delta_i \right)^{k-1} u$$

$^3$The variables ”Resolving Insolvency” on the percent of debt recovered for domestically owned firms in the event of insolvency.

$^4$An example is Chaney (2008)
We use a Gaussian-type kernel density method with bandwidth = 0.1587 where $\kappa$ is a shape parameter that we can calibrate to match the data, and $f(\delta_i) \geq 0$ when $\delta_i \in [0, 1/u]$, and zero elsewhere. In the upcoming sections, it will be convenient to work with the revenue left over after default, that is, the share, $\lambda_{it} = 1 - \delta_i \mu_t$. Working with $\lambda_{it}$ is convenient when we take expectations of future profits because. Employing definition 8, we have

$$E_t(\lambda_{it}) = \lambda_i = 1 - \delta_i u = 1/z_i$$

The PDF for $\lambda_i$ is very simply $f(\lambda_i) = k\lambda_i^{k-1}$. This distribution will yield tractable results in the coming sections.

### 2.8 Firms and Prices

We present a pricing and currency choice component of the model that is similar in spirit to Schmidt-Eisenlohr (2013) and Niepmann and Schmidt-Eisenlohr (2013). In this version, traded firm output is nearly exogenous because we make two simplifying assumptions. First, it is convenient to think of $\lambda_i$ as attached to an infinitesimally small shipment of traded goods, $dC_{t,t+1}$. Therefore, the size of each shipment is fixed at $dC_{T,t+1}$. This essentially forces the allocation of labor across heterogeneous shipments to be exogenous, so that firms allocate labor following the distribution of $\lambda_{it}$, rather than by endogenously following higher profits for a good pull of $\lambda_{it}$. Second, we let the importer always
Figure 4: Solution (Regime 1): When $1 > Z_f / (\hat{Z}_h \tau_I)$

(a) The x-axis are the values of $\lambda$ (home firms), and the y-axis is the value of $\lambda^*$ (foreign firms). Intuitively, the cutoffs make sense. LOCs are used more often as the cost of owning a bank account decreases ($\bar{c}$). Cash-in-Advance is used less often as firms become more reliable ($\kappa$), or the substitutability of home and foreign goods increases ($\sigma$).

have the option of buying from an arbitrager. As there is one traded good, there will then be only one export price. These two assumptions go a long way to keeping results tractable.

Qualitatively, these assumptions do not affect the results. We proceed by characterizing the marginal profits of each payment type. Then, we find the cutoff values of $\lambda_i = 1 - u \delta_i$, above (or below) which the firm will choose a particular pricing type. There are six cutoff values: $\hat{\lambda}_j$ for $j \in \{CIA_h, CIA_f, OAh, OAf, LOC_h, LOC_f\}$. Figures 4 and 5 below show graphically how firms choose different forms of payment, given their level of $\lambda_i$ and the interest rates, $r^h_t$ and $r^f_t$.

In the equations that follow, $\bar{c}$ is the marginal cost per unit of output for holding a bank account, $\tau$ is the percentage cost of exchanging currency, $\tau_I = 1 + \tau$ is the cost to the importer for exchanging currency, and $\tau_E = 1 - \tau$ is the cost to the exporter. We assume that $\bar{c} > 1$, and $\tau > 0$, so that $\tau_I > 1$ and $\tau_E < 1$.

### 2.9 Cash-In-Advance Payments ($CIA_h, CIA_f$)

Under Cash-in-Advance payments, the importer pays for the good one period before it arrives. If the exporter renegs on the contract, they accept payment but do not produce the traded good. Taking first the case where payments are made in home currency, the exporter at home has expected
(a) Similar to above, the x-axis are the values of $\lambda$ (home firms), and the y-axis is the value of $\lambda^*$ (foreign firms). The same intuition applies to this regime.

Profits,

$$E_t \left( \Pi^{EH}_{t+1,CIA} \right) = \left\{ dC^*_{T,t+1} \left( P^E_t - \lambda_t W_{h,t} \right) \right\}$$

Recall that $C_{T,t+1}$ is known at time $t$ because shipments arrive one period late, so that all variables here are known at period $t$.

The foreign importer faces a similar maximization problem. However, the shipment and therefore the revenue arrives one period later. Expected revenues depend on the inflation rate and the exporter’s expected ‘enforceability’ factor, $\lambda_i$. Therefore the importer expects profits that solve

$$E_t \left( \Pi^{IH}_{t+1,CIA} \right) = \left\{ \left( \Delta^*_{t+1} \right) \left( dC^*_{T,t+1} \right) \left( E_t \left[ \frac{\lambda_{t+1,i} P^*_{I,t}}{\Pi^*_{t+1}} \right] - \tau I P_{E,h} \right) \right\}$$

This expression can be simplified using the definition of bond prices, the definition for $\lambda_{t+1,i}$, and by assumption that $\mu_t$ is uncorrelated with stochastic variables. The problem for the importer can be written again in simpler terms as

$$E_t \left( \Pi^{IH}_{t+1,CIA} \right) = \left\{ dC^*_{T,t+1} \left( Z_{t,h} P_{I,t} \lambda_t - \tau I P_{E,h} \right) \right\}$$

As a first step in solving the firms problem, we assume that there are some export prices, $P^E_t$, that exist such that profits are zero for some firm with $\lambda^CIA,EH_{t,t}$. We also assume that there are import prices that exist, $P^I_t$, so that profits are at least zero for some importer with $\lambda^CIA,EH_{t,t}$. We
will then back out the parameters that ensure this equilibrium exists and check that everything is reasonable.\(^5\) Prices that satisfy these assumptions will be,

\[
P_{E,t} = \hat{\lambda}_{CIA,EH} W_{h,t}
\]

\[
P_{I,t} = P_{E,t} \frac{\tau_I}{\hat{\lambda}_{CIA,IH} Q_t Z_f}
\]

Using these prices in firm \(i\)'s profit maximization problem, we arrive at expected profit-per-unit-of-output for the importer and exporter

\[
\frac{dP_{EH}}{dC_{T,t+1}^*} = W_t \left( \hat{\lambda}_{CIA,EH} - \lambda_i \right)
\]

\[
\frac{dP_{IH}}{dC_{T,t+1}^*} = \frac{\tau_I P_t}{Q_t} \left( \frac{\lambda_i}{\hat{\lambda}_{CIA,IH}} - 1 \right)
\]

Naturally, exporters profits are zero when \(\lambda_i = \hat{\lambda}_{CIA,EH}\) and importers profits are zero when \(\lambda_i = \hat{\lambda}_{CIA,IH}\). There will be a region where it is profitable for both firms to price in the Home currency using Cash-In-Advance payments so long as

\[
\hat{\lambda}_{CIA,EH} \geq \hat{\lambda}_{CIA,IH}
\]

If this condition is not satisfied, then profits from Cash-in-Advance transactions will be strictly negative, and firms would never use them. When firms decide to use the foreign currency to settle the transaction, the maximization problem instead yields profits,

\[
\frac{dP_{EF}}{dC_{T,t+1}^*} = W_t \left( \hat{\lambda}_{CIA,EF} - \lambda_i \right)
\]

\[
\frac{dP_{IF}}{dC_{T,t+1}^*} = \frac{P_t}{Q_t} \left( \frac{\lambda_i}{\hat{\lambda}_{CIA,IF}} - 1 \right)
\]

The appendix details the exporter’s and importer’s maximization problem, but they closely follow the ones for \(CIA_h\). Similar to above, firms will only engage in \(CIA_f\) transactions when \(\hat{\lambda}_{CIA,EF} \geq \hat{\lambda}_{CIA,IF}\)

### 2.10 Open Account Payments \((OA_h, OA_f)\)

Open account payments happen when the importer pays the exporter after the delivery. OA transactions are common in international trade, and happen whenever importers have a running account (e.g. accounts payable) with a foreign supplier. The details for the exporter and importer max-

\(^5\)This is done in the on-line appendix for the paper.
mization problem are in the appendix. Taking first the case when payments are made in the home currency, marginal profits, $d\Pi_{OA}^{EH}/dC_{T,t+1}$, and $d\Pi_{OA}^{IH}/dC_{T,t+1}$, are similar to the $CIA_h$ and $CIA_f$ case.

$$\frac{d\Pi_{OA}^{EH}}{dC_{T,t+1}} = W_t \left( \frac{\lambda_i}{\hat{\lambda}_{OA,EH}} - 1 \right)$$

$$\frac{d\Pi_{OA}^{IH}}{dC_{T,t+1}} = \tau_t \hat{Z}_{h,t} P_t^E \left( \hat{\lambda}_{OA,EH} - \lambda_i \right)$$

Both firms earn non-negative profits as long as $\lambda_{OA,IH} > \lambda_{OA,EH}$. If instead payments are made in the foreign currency, the marginal profits become,

$$\frac{d\Pi_{OA}^{EF}}{dC_{T,t+1}} = W_t \left( \frac{\lambda_i}{\hat{\lambda}_{OA,EF}} - 1 \right)$$

$$\frac{d\Pi_{OA}^{IF}}{dC_{T,t+1}} = Z_{f,t} P_t^E \left( \hat{\lambda}_{OA,EH} - \lambda_i \right)$$

Similar to before, firms are willing to use OA foreign currency transactions when $\lambda_{OA,IF} > \lambda_{OA,EF}$.

### 2.11 Letters-of-Credit ($LC_h$, $LC_f$)

In Letters-of-Credit transactions, the banks of the importer and exporter deal directly with one another to ensure payment is sent and received. These transactions are risk-less and the firms profits will not depend on $\lambda_i$ or $\lambda_i^*$. The importer pays a cost that is proportional to the shipment (roughly 0.5%-3% in the data). The exporter pays nothing explicitly, however, the exporter’s bank must be established ‘enough’ to have communications with the importers bank. Importers then pay a cost $f_{tc}$ as a share of the shipment, and exporters pay a small marginal cost, $\bar{c}$, to keep an open account in a bank that mediates LOC traffic. Banks choose $f_{tc}$ to maximize their revenues. When they do, they establish a single export and import price for traded goods. Bank profits are decreasing in the export price, and hump-shaped in the import price, so they will continue to raise $f_{tc}$ until one of two things happens: (1) importing firms have zero profits, (2) the importing firm will choose another payment scheme (e.g. $OA_H$, $OA_F$, etc.). The exporter has expected profits,

$$E_t(\Pi_{t+1,LOC}) = \{ dC_{T,t+1}^* (P_t^E Z_{h,t} - cW_{h,t}) \}$$

The foreign importer faces a similar maximization problem, except they must pay $f_{tc}$ per unit additionally to the bank as a service charge. Note that neither maximization problem involves $\lambda_i$ or $\lambda_i^*$, because the bank takes on all risk associated with $\mu_t$. The foreign $LOC_h$ importer has
expected profits,

\[ E_t(\Pi_{t+1, LOC}) = \left\{ dC^*_{T,t+1} \left(Z_{f,t} P^*_t - \frac{P_{E,h}}{Q_t} \left(\tau_t \hat{Z}_{h,t} + f_{tc}\right)\right) \right\} \]

Import and export prices that are consistent with zero exporter profit will be,

\[ P_t^E = \frac{\bar{c}W_t}{Z_{h,t}} \]  \hspace{1cm} (9)  
\[ P_t^I = \frac{P_t^E}{Z_{f,t}Q_t} (\hat{f}_{tc} + \hat{Z}_{h,t}) \]  \hspace{1cm} (10)

The prices for \( LOC_f \) are similar,

\[ P_t^E = \frac{\bar{c}W_t}{\tau_t Q_t Z_{f,t}} \]  \hspace{1cm} (11)  
\[ P_t^I = P_t^E \left(\frac{\hat{f}_{tc}}{Z_{f,t}} + 1\right) \]  \hspace{1cm} (12)

Marginal profits for \( LOC_H \) and \( LOC_F \) exporters will be zero when \( f_{tc} = \hat{f}_{tc} \). The reasoning follows from our two simple assumptions above. From the first, all firms have a fixed share of output, so exporting firms using\( OA \) or \( CIA \) will raise their export prices as high as possible. Due to arbitrage (our second simple assumption), prices will not exceed the highest marginal export cost. In this case, firms using Letters of Credit have the highest marginal production costs, so export prices will equal the marginal costs of firms using \( LOC_H \) or \( LOC_F \), and profits will be zero for these firms.

Marginal profits for the importer will depend on the value of \( f_{tc} \). This is discussed as part of the solution in the following section.

## 2.12 Solution

By inspecting the profits, we see that firms choose differently depending on the value of \( \hat{Z}_{h,t} \tau_t / Z_{f,t} \).

If this expression is larger than 1, then importers always prefer \( OA_H \) to \( OA_F \). When the expression is smaller than one, the reverse is true. We call these Regime 1 and Regime 2. Firms behave the same across time for the other pricing schemes. For instance, since the cost of exchanging currency is non-negative (i.e. \( \tau_t \geq 1 \)), home exporting firms always prefer \( CIA_H \) to \( CIA_F \). Finally, exporters are indifferent between \( OA_{EF} \) and \( OA_{EH} \), between \( LC_{EH} \) and \( LC_{EF} \), and between \( CIA_{EH} \) and \( CIA_{EF} \). This reasoning is summarized below. Figures 4 and 5 in the appendix show the solutions graphically.

Banks choose \( f_{tc} \) to maximize their trade finance revenues. Firms only choose \( LOC \) when they earn less profits under \( CIA \) or \( OA \) methods. For exporters, we can show that this is always the case.
Summary of Profits

\[ \Pi_{IH}^{OA} \geq \Pi_{IF}^{OA} \quad \text{for } \frac{Z_{h,t} \tau_E}{Z_{f,t}} \geq 1 \quad (\text{Regime 1}) \]

\[ \Pi_{IF}^{OA} < \Pi_{IH}^{OA} \quad \text{for } \frac{Z_{h,t} \tau_E}{Z_{f,t}} < 1 \quad (\text{Regime 2}) \]

\[ \Pi_{EF}^{OA} = \Pi_{EH}^{OA} \quad \text{for } \forall t \]

\[ \Pi_{EF}^{LOC} = \Pi_{EH}^{LOC} \quad \text{for } \forall t \]

\[ \Pi_{EF}^{CIA} = \Pi_{EH}^{CIA} \quad \text{for } \forall t \]

\[ \Pi_{IH}^{CIA} \geq \Pi_{IF}^{CIA} \quad \text{for } \tau_I \geq 1 \]

\[ \Pi_{IH}^{CIA} < \Pi_{IF}^{CIA} \quad \text{for } \tau_I < 1 \]

whenever \( \lambda_i < \lambda_{CIA,HI}^{CIA,HI} \). Likewise for importers, we can show that this is always the case whenever \( \lambda_i < \lambda_{OA,HE}^{OA,HE} \).

Under Regime 1, banks maximize their revenue \( R_t(f_{tc}) \). Revenues are the product of the trade interest rate, times the share of firms using either \( LOC_H \) or \( LOC_f \). In Regime 1,

\[ R_t(f_{tc}) = \max_{f_{tc}} \left\{ f_{tc} C_{T,t+1} P_t^E F_L \left( \hat{\lambda}_{CIA,HI}^{CIA,HI} \right) F_L^* \left( \hat{\lambda}_{OA,HE}^{OA,HE} \right) \right\} \quad (13) \]

Using Equations 6 and 9 in this equation, the bank will choose \( f_{tc} \) so that profits are zero for importers under \( LOC_H \), and final import prices will be

\[ P_t^I = \left( \frac{\sigma + \kappa}{\sigma + \kappa - 1} \right) \frac{\bar{c}W_t \hat{Z}_{h,t} \tau_I}{Z_{t,h} Z_{t,f} Q_t} \quad (14) \]

Under Regime 2, banks again maximize their revenue, but now trade finance is contending with \( OA_F \) and \( CIA_F \). Therefore the CDF’s in Equation 13 are evaluated at \( \lambda_{CIA,FI}^{CIA,FI} \) and \( \lambda_{OA,FE}^{OA,FE} \). Banks now choose \( f_{ct} \) so that imported goods prices are,

\[ P_t^I = \left( \frac{\sigma + \kappa}{\sigma + \kappa - 1} \right) \frac{\bar{c}W_t}{\tau_E Z_{t,f} Q_t} \quad (15) \]

The corresponding interest rates for trade finance are

\[ \text{Regime 1: } f_{tc,t} = \frac{\hat{Z}_{h,t} \tau_I}{\sigma + \kappa - 1} \]

\[ \text{Regime 2: } f_{tc,t} = \frac{Z_{f,t}}{\sigma + \kappa - 1} \]

These will satisfy the zero-profit conditions for the Importer (Equations 12 and 10). The Regime with higher \( f_{ct} \) will be the one that banks prefer because it yields higher profits. Therefore, like firms, banks prefer transactions in home currency when \( \hat{Z}_{h,t} \tau_E / Z_{f,t} \geq 1 \). They will offer an infinitesimally small amount of profit to firms in order to encourage a contract written in home currency.
Altogether, when $1 \leq \tau_I$, $1 < \bar{c}$, and $\kappa$ and $\sigma$ are within a reasonable range,$^6$ firms choose:

**Proposition 1**: Under Regime 1, when $\hat{Z}_{h,t} \tau_E/Z_{f,t} \geq 1$, firms use three pricing strategies: \{\Pi_{CIA}^{IH}, \Pi_{OA}^{IH}, \Pi_{LOC}^{IH}\}. See Appendix 3 for the proof.

**Proposition 2**: Under Regime 2, when $\hat{Z}_{h,t} \tau_E/Z_{f,t} < 1$, firms use three pricing strategies: \{\Pi_{CIA}^{IH}, \Pi_{OA}^{IF}, \Pi_{LOC}^{IF}\}. See Appendix 3 for the proof.

### 2.13 Closing the Model

Aggregating across payments for all exporting firms is straightforward. The model is constructed so that payments for LOC and OA transactions are lagged one period. These delays leave (potentially) large income flows vulnerable to changes in inflation or the real-exchange rate. Also, as constructed, CIA flows are forward looking, because they are payments for goods not yet consumed. The reader should note that payments on Open Account transactions are factored by the percent of shipments that are successfully sent, or the average $\bar{\lambda}$ over the regions of where firms choose OA pricing. Each $\bar{\lambda}_j$ is defined as the average $\lambda$ of the region where firms choose pricing option $j$, and then multiplied by probability of finding a firm in region $j$. That is, $\bar{\lambda}_j = \int_{\lambda \in j} \lambda dF(\lambda)$.

The $\eta_{j}^{\star}$ variables are the shares of traded consumption sent via payment type $j$. For example, $\eta_{CIA}^{\star} = \int_{\lambda \in CIA,EH} \lambda dF(\lambda)$.

Imports have a similar look, where again they are summed across the six types payments. Demand is now domestic (i.e. without a star). Also, this includes payments to the international bank in the form of Letters-of-Credit, so there is an extra term at the end of the expression. We assume the bank is headquartered in the foreign country, so LOC fee payments are counted as strictly a loss.

**Imports**

\[
M_t = P_{t,E}^{\star} C_{T,t+1}^{\star} \left( \eta_{t}^{\star CIA} + \eta_{t}^{\star CIAf} \right) + \frac{P_{t-1,E}^{\star} Q_t}{\Pi_t} C_{T,t}^{\star} \left( \bar{\lambda}_{t-1}^{OA} + \eta_{t-1}^{\star LCh} \right) + \frac{P_{t-1,E}^{\star} C_{T,t}^{\star} Q_t}{\Pi_t} \left( \bar{\lambda}_{t-1}^{OAf} + \eta_{t-1}^{OAf} \right) + f_{TCH}^{\star} C_{T,t+1}^{\star} P_t^{\star} \left( \eta_t^{\star LCh} + \eta_t^{\star LC} \right)
\]

**Exports**

\[
X_t - M_t = Z_{f,t} B_{t+1}^{\ell} Q_t - \frac{B_{t}^{\ell} Q_t}{\Pi_t^{\star}}
\]

---

$^6$Having all $CIA$, $LOC$ and $OA$ present at the same time requires that $\frac{(\sigma + \kappa - 1)}{(\sigma + \kappa)} < \hat{Z}_{h,t}$ in Region 1, and $< \hat{Z}_{h,t}$ in Region 2.
2.14 Summary of Model Solution

An equilibrium for this model will be, for each $t$, consumers optimally choose $L_t, L^*_t, P^{xf}_t, Z_{h,t}, Z_{f,t}, B_{f,t+1}, \tilde{M}_t, C_t, C_{T,t+1}, C_{N,t}, Q_t, P^I_t, P^E_t, f_{t,c,t}$, and shares $\eta^j_t$ for each $j$ in pricing type, taking as given two stochastic shocks to foreign inflation and consumption, $\epsilon^c_t$ and $\epsilon^\pi_t$, and seven predetermined variables: $Q_{t-1}, C^*_t, \Pi^*_t, B_{f,t}, \tilde{M}_t, S^B_t, S^B_{t-1}$, in order to satisfy the 12 equations in Part 2 of Table 1, and the five equations of Part 1 of Table 1 Regime 1 when $\hat{Z}_{h,t}\tau_E/Z_{f,t} \geq 1$, and Table 1 Regime 2 whenever $\hat{Z}_{h,t}\tau_E/Z_{f,t} < 1$. The full summary of all model equations is in Table 1. We define $S^A_{t-1}$ and $S^B_{t-1}$ as the sum of the lagged components of net exports (Equations 16) in home and foreign currency, respectively.
## Table 1: Model Summary

**Part 1**

<table>
<thead>
<tr>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{t,c,t} = \frac{Z_{h,t}^\tau_t}{\sigma + \kappa - 1}$</td>
<td>$f_{t,c,t} = \frac{Z_{f,t}}{\sigma + \kappa - 1}$</td>
</tr>
<tr>
<td>$P_t^E = \frac{\bar{c} W_t}{Z_{t,h}}$</td>
<td>$P_t^E = \frac{\bar{c} W_t}{Z_{t,f} \tau^E}$</td>
</tr>
<tr>
<td>$P_t^I = \left( \frac{\sigma + \kappa}{\sigma + \kappa - 1} \right) \frac{\bar{c} W_t \hat{Z}<em>{h,t}^\tau_t}{Z</em>{t,h} Z_{t,f} Q_t}$</td>
<td>$P_t^I = \left( \frac{\sigma + \kappa}{\sigma + \kappa - 1} \right) \frac{\bar{c} W_t}{\tau^E Z_{t,f} Q_t}$</td>
</tr>
<tr>
<td>$\eta_{t,OA}^H = \left( 1 - \left( \frac{1}{\bar{c}} \right) \kappa \right) \left( \frac{\sigma + \kappa - 1}{Z_{t,h} \left( \sigma + \kappa \right)} \right)^\kappa$</td>
<td>$\eta_{t,OA}^F = \left( 1 - \left( \frac{1}{\bar{c}} \right) \kappa \right) \left( \frac{\sigma + \kappa - 1}{Z_{t,f} \left( \sigma + \kappa \right)} \right)^\kappa$</td>
</tr>
<tr>
<td>$\eta_{t,CIA}^H = 1 - \left( \frac{\kappa}{Z_{h} \left( \sigma + \kappa \right)} \right)^\kappa$</td>
<td>$\eta_{t,CIA}^F = 1 - \left( \frac{\tau^I \left( \sigma + \kappa - 1 \right)}{Z_{h} \left( \sigma + \kappa \right)} \right)^\kappa$</td>
</tr>
<tr>
<td>$\eta_{t,LC}^H = 1 - \eta_{t,OA}^H - \eta_{t,CIA}^H$</td>
<td>$\eta_{t,LC}^F = 1 - \eta_{t,OA}^F - \eta_{t,CIA}^F$</td>
</tr>
</tbody>
</table>

**Part 2**

Equations Consistent Across Regime Types

\[
X_t - M_t = Z_{f,t} B_{t+1}^I Q_t - B_{t}^I Q_t
\]

\[
C_t = \left( \nu^{1/\sigma} \left( C_{T,t} \right)^{(\sigma-1)/\sigma} + \left( 1 - \nu \right)^{1/\sigma} \left( C_{N,t} \right)^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}
\]

\[
Z_{h,t} = 1 - \varrho \left( \tilde{M}_t \right)^{-\epsilon} C_t^\gamma
\]

\[
Z_{h,t} = \beta \bar{E}_t \left( \Lambda_{t+1} / \Pi_{t+1} \right)
\]

\[
Z_{f,t} = \beta \bar{E}_t \left( \Lambda_{t+1}^* / \Pi_{t+1} \right)
\]

\[
Z_{f,t} = \beta \bar{E}_t \left( \left( \Lambda_{t+1} / \Pi_{t+1} \right) Q_{t+1} / Q_t \right)
\]

\[
C^*_{T,t+1} = \nu C_t^{\gamma \sigma} \left( P^*_{t,t} Z_{t,f} / E_t \left( C_{t+1}^{1/\sigma - \gamma} \right) \right)^{-\sigma}
\]

\[
C^*_{T,t+1} = \nu C_t^{\gamma \sigma} \left( P_{t,t} Z_{t,h} / E_t \left( C_{t+1}^{1/\sigma - \gamma} \right) \right)^{-\sigma}
\]

\[
L_t = C_{N,t} + C_{T,t} \left( \tilde{\lambda}_{t}^{CIAh} + \tilde{\lambda}_{t}^{CIAf} \right)
\]

\[
L^*_t = C^*_{N,t}
\]

\[
C_t^* = \rho_{e^*} \left( C_{t-1} - \bar{C}^* \right) + e_t^e
\]

\[
\Pi_t^* = \rho_{\Pi^*} \left( \Pi_{t-1}^* - \bar{\Pi}^* \right) + e_t^\Pi
\]
3 Solution Method

The model is solved using a Paramaterized Expectations Algorithm (PEA) method that is similar to the procedure in Kubler and Schmedders (2003) and described in more general terms in Fackler (2004). The results using this global solution method are close to a third-order local approximation using Dynare, however, we take the extra step solving it globally for several reasons. First, as pointed out by Rabitsch, Stepanchuk, and Tsyrennikov (2015), the long term distribution of assets may be quite different from the approximated NFA positions derived using a local-approximation method, such as the one in Devereux and Sutherland (2011). For most parameters, the NFA distribution in the global solution is similar to the local solution.

Second, the complete model has a discontinuity as it transitions from Regime 1 to 2. A global solution method is able to accommodate discrete jumps, whereas perturbations solutions methods rely on derivatives, and therefore can only manage differentiable equations.

Third, the steady-state and non-stochastic steady state are not generally the same. Linearizing around the wrong steady state and deriving results is a concern. There are several papers that offer work-arounds with revised perturbation techniques, such as Juillard (2011), Coeurdacier, Rey, and Winant (2011), Gertler, Kiyotaki, and Queralto (2011), de Groot (2013), but these are not widely used and often impractical to implement.

Fourth, we would like to avoid having to use an Endogenous Discount Factor (EDF) to guarantee stability of the NFA position. Some mechanism, such as an EDF or asset transaction costs, is necessary to keep the NFA distribution stable. Stepanchuk and Tsyrennikov (2015) points out, however, that using an EDF or having transaction costs succeeds in stabilizing the NFA, but it does so in a ‘very exogenous way.’ The parameters chosen in the EDF will dominate the shape of the final NFA distribution.

4 Results

Referring to Figures 4 and 5, a key model prediction is that countries with higher costs for enforcing contracts will tend to pay early for imported goods, and thus be creditors rather than debtors of trade credit. This first result falls out rather mechanically from our model, coming from the fact that borrowing costs are higher for untrustworthy firms due to a risk premium. In expectation, prices charged to firms with lower values of $\lambda_i$ must be higher so that, on average, sellers are compensated for their losses from reneging. The literature around trade credit, as in Ahn et al. (2011), Ahn (2014), and Schmidt-Eisenlohr (2013), proposes similar mechanisms to ours in order to yield similar predictions regarding which types of firms extend credit. We too show that more trustworthy agents can borrow when trading. In the model, this result can be seen by looking to
Figures 4 and 5: as $\kappa$ increases and the home country firms becomes more trustworthy, the share of firms choosing Open Account and Letter of Credit financing increases. In other words, home country firms are extended trade credit. This follows because $\frac{d}{d\kappa} \left( (1 + r)^\frac{\kappa + \sigma - 1}{\kappa + \sigma} \right) > 0$ for positive values of $\kappa$ and $\sigma$.

This prediction from the model is readily seen in the data. Table 2 divides a sample of 30 countries into net creditors and net debtors of trade credit. We then compare three measures of contract enforcement pulled from the World Bank’s Doings Business Survey for the sample of countries. In the sample, firms that are net-creditors are generally inhabitants of countries where it is costly to enforce a contract. For example, if a domestic firm defaults on a creditor country, on average only 40% of the firms assets are recoverable. By comparison, in countries where trading firms pay late, twice the share of assets are recoverable and enforcements costs as a percent of the contract are 4% lower. A full description of the data for Figure 2 is in the Appendix. One novel approach about these results is that, while the literature often uses country specific surveys, it is also possible to get an estimate for the stock of trade credit by cumulating the Balance of Payments transactions.\footnote{See Section 3.1.2.2 of the Balance of Payments Manual gives details for the appropriate line items for non-governmental cross border transactions.} This is the approach we use in Chapter 1 of this dissertation, and we use it here as well to find gross positions on trade credit.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Debtors</th>
<th>Creditors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost to enforce contract (% of Contract)</td>
<td>17</td>
<td>21</td>
</tr>
<tr>
<td>Un-recovered if insolvency (% of total)</td>
<td>18</td>
<td>41</td>
</tr>
<tr>
<td>Days to resolve disputes</td>
<td>395</td>
<td>496</td>
</tr>
<tr>
<td>Net Assets (Foreign Currency % of GDP)</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>Net Assets (Home Currency % of GDP)</td>
<td>-30</td>
<td>-40</td>
</tr>
<tr>
<td>Net Assets (Total % of GDP)</td>
<td>-13</td>
<td>-30</td>
</tr>
</tbody>
</table>

In the model, as the degree of contract enforcement rises, the country’s net foreign asset position falls. In the numerical solution of the model shown in 3, we simulate bond holdings in the home country for values of the enforcement costs, $\kappa$, which we plot in Figures 6 and 7. This holds true for both Regime 1 when $r_h^t > r_f^t$, and in Regime 2 when $r_h^t < r_f^t$. A first order reason for this result is the following: with higher degrees of contract enforcement, buying from abroad or selling abroad is less expensive because the markup on foreign goods is lower. Therefore consumers substitute towards foreign goods and they use foreign financing to support their new consumption basket. This first-order effect accounts for the rightward shift of the bell-shaped distributions in 6, and 7, as the curves move from red to blue. A second order effect also drives the numerical results in the model. By comparing the foreign assets in Figure 6 and 7 for any given level of $\kappa$, the plots
While in Regime 1 (with all exports in home currency), we calibrate the Pareto shape parameter $\kappa$ to varying degrees of intensity. Higher values signify smaller losses during default. We show here that a country’s precautionary savings increases as $\kappa$ decreases. In Regime 1 are shifted to the right so that home lending is higher at every level of $\kappa$. When all trade is the home currency, since the home country is only able to borrow from abroad in units of the foreign currency, the importer uses trade credit to get around the incomplete markets and effectively hedge the real exchange rate risk through trade. This argument is reminiscent of Cole and Obstfeld (1991), but the model’s structure is quite different. Here, as the risk-averse importer invests in home-currency denominated traded goods, that pay off next period is in units of the home good. By borrowing in the foreign currency and purchasing assets that pay off in domestic currency, the home country is hedging the risk that comes through their balanced-trade equation (16).

The mechanism just described may be a force in the data, but it is probably a small one. There are at least two reasons why the scenario we just described is a nice thought experiment but may not present itself in the data. First, in the model we assumed that a country cannot borrow in domestic currency with bonds, but they can borrow through trade credit. Realistically in the data, the ability to borrow in securities markets in domestic currency, and the ability to denominate traded goods in domestic currency, are highly correlated. Trade in our model allows the home country to hedge risk currency—but in the data, it is more likely that hedging risk with trade credit is plagued with frictions more so than through the bond markets. Second, even if this is a valid channel, it is likely overshadowed by the other global drivers identified in Lane and Milesi-Ferretti (2007) and Lane and Shambaugh (2010). For example, looking again to Figure 2, creditors of trade credit tend to be heavy borrowers from abroad, which is an observation that runs directly counter to the model’s prediction. Rather, the model in this paper is highly stylized and meant to focus on the effects of trade credit. Much of the standard machinery in DSGE models, such as capital stocks, sticky prices,
Figure 7: Assets with Varying levels of Contract Enforcement (Regime 2)

(a) While in Regime 2 (with OA and LOC exports in the foreign currency), the precautionary savings motive increases, as does the volatility of asset holdings.

or government assets, are not present to generate realistic predictions about net asset positions.

Even so, there are two predictions of this model that specifically pertain to trade credit and they are both readily testable in the data. We show below that this data is favorable towards our model. First, the model predicts that changes in the interest rate can be a dominant factor that determines the share of home currency denominated trade credit. Looking to Figures 4 and 5, the only dynamic variables that determine trade credit are the interest rates. In Regime 1, as the home interest rate rises, trade credit in home currency falls. In Regime 2, as the home rates rise, all trade remains in foreign currency and there is no change. In total then, a rise in home rates relative to foreign will always lead to a fall in home-currency trade credit, or at least no-change. In the first four columns of Table 3, we present the results of regressing the changes in the interest rate spreads on the changes in the log-share of home currency denominated trade credit. Home interest rates are defined as three-month interest return on government securities, while the ‘foreign interest rate’ is calculated as the trade weighted average of interest rates across all trading partners. As expected, the signs on the estimated coefficients are negative and significant. Although the $R^2$ values are low for import-weighted interest rates, they are surprisingly high for the export-weighted ones.

A second testable prediction of the model regards total behavior of trade credit. In particular, when $r_f - r_h < 0$, then trade credit should fall when foreign interest rates rise. When the opposite is true, and $r_f - r_h > 0$, then trade credit should fall with an increase in the home interest rate. Using changes in trade credit as the dependent variable, and the home and foreign interest rates as explanatory variables, we show in Column 5 of Table 3 that the behavior of trade credit weakly accords with the model’s predictions—the signs on coefficients in Column 5 are negative, and at least one of the coefficients is significant. What is the intuition behind this result? Importers and exporters are sensitive to the opportunity cost of trade credit. As interest rates rise in the currency
Table 3: Regression Results

<table>
<thead>
<tr>
<th></th>
<th>Dependent variable:</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>log(Share of Imports)</td>
<td>log(Share of Exports)</td>
<td>d(Trade Credit)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>Exports Weighted:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r^h - r^f$</td>
<td>-0.126***</td>
<td>-0.085***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.022)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Imports Weighted:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r^h - r^f$</td>
<td>-0.120***</td>
<td>-0.083***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.022)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(I_{r^f &gt; r^h &lt; 0}) \cdot r^f$</td>
<td></td>
<td></td>
<td></td>
<td>-0.310***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.071)</td>
<td></td>
</tr>
<tr>
<td>$(1 - I_{r^f &gt; r^h &lt; 0}) \cdot r^h$</td>
<td></td>
<td></td>
<td></td>
<td>-0.083</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.098)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.332***</td>
<td>-1.328***</td>
<td>-2.076***</td>
<td>-2.076***</td>
<td>-0.127</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.055)</td>
<td>(0.046)</td>
<td>(0.046)</td>
<td>(0.104)</td>
</tr>
<tr>
<td>Observations</td>
<td>609</td>
<td>609</td>
<td>609</td>
<td>609</td>
<td>1,597</td>
</tr>
<tr>
<td>R²</td>
<td>0.158</td>
<td>0.157</td>
<td>0.541</td>
<td>0.540</td>
<td>0.012</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.121</td>
<td>0.120</td>
<td>0.520</td>
<td>0.520</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Note: *p<0.1; **p<0.05; ***p<0.01. All regressions include country fixed effects, and the annualized average U.S. interest rate as controls. Trade weighted interest rates are weighted by total trade.

of invoicing, so does the opportunity cost of holding non-interest bearing debt in that currency. The overall weakness of the model’s explanatory power is sobering, but as we showed in Chapter 1, there are many other factors that can potentially explain the dynamics of trade credit over the business cycle. For instance, during sudden stops, we showed that much of trade credit fluctuations are explained by leverage constraints and changes in the nominal exchange rate.

5 Conclusion

In this paper we develop a currency and payment choice model within a two-country dynamic general equilibrium framework. Most international trade is settled in one of three ways. Perhaps most commonly used are Open Account transactions, whereby the importer pays the exporter after the good arrives. Secondly, there are Letters of Credit transactions, where the importer’s bank pays the exporter’s bank upon arrival of the good. Finally, firms can require a Cash-in-Advance transaction when the importer pays for the product before it arrives. We derive simple closed form solutions for the share of firms that use each settlement method, and also the preferred currency of the transaction. Finally, we show how net exports are a sum of pricing contracts: one forward looking, one backward looking and one contemporaneous, each with a different exposure to interest and exchange rate fluctuations.
The two most pressing ways to expand on this paper are the following: first, to develop the empirical portion of the paper and test the precautionary savings predictions; second, we must relax our ‘simple model’ assumptions about traded firm production. We assume in this model that when firms trade with each other, they always ship the same infinitesimally small amount. This is done to arrive at clean closed form solutions, but at the least, these should be relaxed in the numerical portions of the paper. Among other extensions, it would be interesting to apply similar contracting frictions to the Non-traded goods sector. Delays in deliveries of non-traded goods are shorter, but perhaps relevant to the conclusions.

### A Data

*Currency Invoicing:* All data on currency of invoicing in international trade follows from the datasets we created in Chapter 1 of this dissertation. Please see Table (?) in the Appendix (?) for details about the two distinct datasets we use.

*Table 2:* All data on trade credit and contract enforcement is from 1995 until 2015. Table reports the simple averages ac we take averages for each country over the full time period. The countries in this data sample are the following: Australia, Austria, Belgium, Brazil, Canada, Czech Republic, Germany, Denmark, Spain, Estonia, Finland, France, United Kingdom, Greece, Hungary, Ireland, Italy, Japan, Korea, Republic of, Lithuania, Netherlands, Norway, Poland, Portugal, Slovakia, Slovenia, Sweden, Thailand, United States.

### B Proofs

The body of the paper outlines the rationale behind Propositions 1 and 2. Proofs are available in the appendix here: https://www.w-swanson.com/

**Cash-In-Advance in Foreign Currency** \((CIA_f)\) If foreign currency is used instead as the export currency, the maximization problem becomes

\[
\begin{align*}
\text{Exporter}: \quad & \max_{C_{T,t+1}} \left\{ (\Delta_{t+1}) \left( \tau_{E} Q_{t} C_{T,t+1}^{*} P_{CIA_{f}}^{E,E,t} - \lambda_{i,t} W_{h,t} C_{T,t+1}^{*} \right) \right\} \\
\text{Importer}: \quad & \max_{C_{T,t+1}} \left\{ \left( C_{T,t+1}^{*} Z_{t,f} P_{I,t} \lambda_{i} - \tau_{I} C_{T,t+1}^{*} P_{E,t} \right) \right\}
\end{align*}
\]
The prices that satisfy zero profits here are

\[ P_{E,t}^{ClAf} = \frac{\lambda_{i,t} W_{h,t}}{\tau_I} \]  

(19)

\[ P_{I,t}^{ClAf} = \frac{P_{E,t}^{ClAf}}{\lambda_{i,t} Z_f} \]  

(20)

**Open Account in Home Currency (OA\(_h\))** The exporter and importer problems are,

Exporter: \( \max_{C_{T,t+1}} \{ Q_t C_{T,t+1}^{*} P_{E,t}^{ClAf} - \lambda_{i} W_{h,t} C_{T,t+1}^{*} \} \)  

(21)

Importer: \( \max_{C_{T,t+1}} \{ C_{T,t+1}^{*} Z_{t,f} P_{I,t} - \tau_I \lambda_{i} Z_{t,f} C_{T,t+1}^{*} P_{E,t} \} \)  

(22)

The prices that satisfy zero profits here are

\[ P_{E,t}^{OAh} = \frac{W_{h,t}}{\lambda_{i,t} \tau_I Z_h} \]  

(23)

\[ P_{I,t}^{OAh} = \frac{P_{E,t}^{OAh} \lambda_{i,t} Z_{h,t} \tau_I}{Q Z_{f,t}} \]  

(24)

**Open Account in Foreign Currency (OA\(_f\))** If foreign currency is used instead as the export currency, the maximization problem becomes

Exporter: \( \max_{C_{T,t+1}} \{ Z_{h,t} C_{T,t+1}^{*} P_{E,t}^{OAf} - c W_{h,t} C_{T,t+1}^{*} \} \)  

(25)

Importer: \( \max_{C_{T,t+1}} \{ C_{T,t+1}^{*} Z_{t,f} P_{I,t} - \tau_I \lambda_{i} Z_{t,f} C_{T,t+1}^{*} P_{E,t} \} \)  

(26)

The prices that satisfy zero profits here are

\[ P_{E,t}^{OAf} = \frac{W_{h,t}}{\lambda_{i,t} \tau_E Z_f Q_t} \]  

(27)

\[ P_{I,t}^{OAf} = P_{E,t}^{OAf} \lambda_{i,t}^{*} \]  

(28)

**Letters-of-Credit in Home Currency (LC\(_h\))** If foreign currency is used instead as the export currency, the maximization problem becomes

Exporter: \( \max_{C_{T,t+1}} \{ Z_{h,t} C_{T,t+1}^{*} P_{E,t}^{LCh} - c W_{h,t} C_{T,t+1}^{*} \} \)  

(29)

Importer: \( \max_{C_{T,t+1}} \{ C_{T,t+1}^{*} Z_{t,f} P_{I,t} - \frac{C_{T,t+1}^{*} P_{E,t}}{Q_t} (\tau_I Z_{t,f} + f_{tc,t}) \} \)  

(30)
The prices that satisfy zero profits here are

\[ P_{E,t}^{LCh} = \frac{cW_{h,t}}{Z_h} \]

\[ P_{I,t}^{LCh} = \frac{P_{E,t}}{Z_{f,t}Q_t} \left( f_{c,t} + \hat{Z}_{h,t}\tau_I \right) \]

**Letters-of-Credit in Foreign Currency** \((LC_f)\) If foreign currency is used instead as the export currency, the maximization problem becomes

Exporter: \( \max_{C_{T,t+1}} \left\{ Z_{f,t}C_{T,t+1}^*P_{E,t}^{LC_f}Q_t\tau_E - cW_{h,t}C_{T,t+1}^* \right\} \) \hspace{1cm} (31)

Importer: \( \max_{C_{T,t+1}} \left\{ C_{T,t+1}^*Z_{t,f}P_{I,t} - C_{T,t+1}^*P_{E,t}(Z_{t,f} + f_{c,t}) \right\} \) \hspace{1cm} (32)

The prices that satisfy zero profits here are

\[ P_{E,t}^{LC_f} = \frac{cW_{h,t}}{Z_{f,t}\tau_E} \]

\[ P_{I,t}^{LC_f} = P_{E,t} \left( \frac{f_{c,t}}{Z_{f,t}} + 1 \right) \]

**References**


Coeurdacier, N., Rey, H., & Winant, P. (2011). Solving optimal portfolio models around a risky steady state. In *Assa annual meeting, january* (pp. 6–9).


